Abstract
Commodity terms of trade shocks have continued to drive macroeconomic fluctuations in most emerging market economies. The volatility and persistence of these shocks have posed great challenges for monetary policy. This study employs a New Keynesian Dynamic Stochastic General Equilibrium (DSGE) model to evaluate the optimal monetary policy responses to commodity terms of trade shocks in emerging market economies. The model is calibrated to the South African economy. The study shows that CPI inflation targeting performs relatively better than exchange rate targeting and non-traded inflation targeting both in terms of reducing macroeconomic volatility and enhancing welfare. However, macroeconomic stabilisation comes at a cost of increased exchange rate volatility. The results suggest that the appropriate response to globally induced exogenous shocks is to adopt inflation targeting. But the central bank need to pay attention to the country's exposure to exchange rate fluctuations.

Keywords: Commodity terms of trade, monetary policy; DSGE

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The views expressed in this paper are the authors and not necessarily those of their respective institutions
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1 Introduction

Commodity terms of trade shocks have continued to shape the macroeconomic dynamics of most emerging market economies (EMEs) for many years (Mendoza, 1995). These disturbances have proved to be very volatile and persistent, especially in commodity exporting countries, resulting in significant macroeconomic volatility. In the face of such shocks, the challenge for policy makers is to find appropriate policies to dampen the shocks and stabilise the economies rather than amplify the shocks. In this context, what is the appropriate monetary policy response to commodity terms of trade shocks?

Several studies have analysed the macroeconomic implications of alternative monetary policy regimes following domestic and external shocks in small open economies (see e.g. Gali and Monacelli, 2005; Medina and Soto, 2005; Devereux et al., 2006; Blanchard and Gali, 2007). Although these shocks can induce some macroeconomic volatility in small open economies, little attention have been given to the analysis of commodity terms of trade shocks and their implications for monetary policy design in emerging market economies. Yet, as Chen and Rogoff (2003) point out, commodity prices are the most volatile and persistent component of the overall terms of trade. As such, they can generate more substantial movements in exchange rates and other macroeconomic variables. Also, the consideration of commodity terms of trade shocks under sticky prices may significantly change the conventional wisdom on optimal policy in EMEs (Obstfeld and Rogoff, 2000).

In terms of policy responses especially to external shocks, much prominence was given to the role of flexible exchange rates (see e.g. Friedman, 1953; Chia and Alba, 2006). But flexible exchanges without activist monetary policy may not adjust in the right direction to achieve the desired outcomes, resulting in negative welfare effects (Devereux, 2004). Therefore, policy responses should go beyond exchange rate choice and should also consider monetary policy responses. Indeed, recent studies have shown that monetary policy such as inflation targeting can play a role in dampening cyclical macroeconomic fluctuations and improve welfare in small open economies (see e.g. Svensson, 2000; Cuche-Curti et al., 2008; Orphanides, 2008). But the question that remains is which monetary policy regime delivers better macroeconomic outcomes in the face of commodity terms of trade shocks.

This study develops a New Keynesian dynamic stochastic general equilibrium (DSGE) to analyse the appropriate monetary policy framework in a commodity de-
dependent economy. Precisely, the study evaluates the relative merits of CPI inflation targeting (CIT) rule compared with non-traded inflation targeting (NTIT) rule and exchange rate targeting (ET) rule in the face of commodity terms of trade shocks. Within the same framework, the study also examine the monetary policy implications of productivity shocks in the commodity sector. The model is framed in the new open economy macroeconomic (NOEM) framework which integrates features such as nominal rigidities and monopolistic competition.\textsuperscript{1} It builds closely on the work of Devereux et al. (2006), Gali and Monacelli (2005) and Cashin et al. (2004) and is modified to take into account the commodity sector. The incorporation of the commodity sector allows the explicit examination of commodity terms of trade shocks and their implications for optimal monetary policy. The framework developed by Cashin et al. (2004) is used to capture the commodity terms of trade in a broader framework of a monetary model.

The model is calibrated to the South African economy. South Africa is an ideal country for this analysis because it has a significant portion of its trade concentrated in primary commodities such as gold, platinum and diamond. Cashin et al. (2004) considers the South African rand as a commodity currency because of its high volatility and sensitivity to the movement in commodity prices. Further, Aron et al. (2000) argues that South Africa’s overall terms of trade have been mostly influenced by fluctuations in the world price of its key commodity exports.

This study contributes to literature in a number of ways. First, it constructs a multi-sector DSGE model to characterise the optimal monetary policy of a small open commodity dependent emerging market economy. Several factors motivate the choice of a DSGE model for this analysis. For instance, the analytical power and coherence of DSGE models in policy analysis help to address a number of issues around exogenous shocks and welfare in the presence of sticky prices. More importantly, DSGE models are micro-founded in the sense that they are explicitly derived from the constrained optimising behavior of households and firms in the economy (Tovar, 2008). Further, they are structural in nature, which permits clear identification, interpretation and discussion of alternative policy interventions and their transmission mechanisms (Smets and Wouters, 2003). Finally, as argued by Woodford (2003), DSGE models help to overcome the Lucas critique because the estimated deep structural parameters are less likely to change when policies change.

Second, the model features the commodity sector. This characterisation espe-

\textsuperscript{1}See Lane (2001) for a detailed survey of the new open economy macroeconomics.
cially in a dynamic equilibrium setting is not so unique to many small open economy models. This enables the examination of whether and how the introduction of the commodity sector change the conventional wisdom on optimal monetary policy in emerging market economies. The model is simulated in order to analyse the dynamic responses of macroeconomic variables to commodity shocks under different monetary policy rules. The study then demonstrate that the model can closely mimic the empirical regularities of the transmission processes of commodity terms of trade shocks in EMEs.

Thirdly, using the central bank loss function, the study evaluates and compares the welfare implications of alternative monetary policy regimes to determine the optimal monetary policy in countries prone to commodity shocks. Thus, it provides guidance to the formulation of monetary policy in emerging market economies. The loss function is chosen for welfare evaluation because it is relatively tractable and flexible and allows the derivation of closed form solutions of the endogenous variables of the model.

The analysis shows that commodity terms of trade shocks induce lower responses of some macroeconomic variables under CIT than under NTIT and ET. However the stabilisation of the economy by CIT comes at the expense of high real and nominal exchange rate fluctuations. Even when productivity shocks in the commodity export sector are considered, real and nominal exchange rates fluctuate more under CIT than in other regimes. These results are generally robust to several sensitivity tests on key parameters which include trade openness, price stickiness and elasticity of substitution of traded and non-traded goods.

The study also evaluates the volatility and welfare implications of alternative monetary policy rules. It shows that the economy achieves less volatility in aggregate and sectorial output, consumption and CPI inflation under CIT rule while NTIT rule delivers less volatility in non-tradable prices and non-tradable inflation. In terms of welfare, the CIT rule delivers less welfare losses than other rules when the central bank prefers more inflation and interest rate stabilisation. In this case, the NTIT rule delivers welfare outcomes that are lower but close to the CIT rule, while the ET rule performs worst. However, when the central bank cares more about output stabilisation, then it achieves less welfare losses by targeting NTIT while ET performs better when central bank increases preference towards exchange rate stabilisation.

The rest of the paper is structured as follows. Section 2 provides some review of related literature. Section 3 develops the model while section 4 describes the
calibration of parameters. Section 5 provides the analysis and interpretation of the results. Section 6 provides the sensitivity analysis. Finally, section 7 concludes and provides policy recommendations.

2 Related Literature

There are many studies that have examined the design of monetary policy in small open economies using DSGE models. Much previous work has focused on the analysis of optimal monetary policy rules in the face of several shocks. Given the diverse conclusions and the specificity of focus of individual studies, it is important to briefly review some relevant studies.

Laxton and Pesenti (2003) construct a small open economy model to investigate the stabilising properties of different monetary policy rules under various shocks. They find that inflation-forecast-based rules perform better than conventional Taylor rules in stabilising the variability of output and inflation in small open emerging economies. In a related study, Parrado (2004) develops a DSGE model to explain how differences in monetary policy and exchange rate regimes account for macroeconomic volatility and welfare in a small open economy. He finds that domestic inflation targeting yields better outcomes than CPI targeting since the former can stabilise both inflation, output gap and real exchange rate. He also observes that in the face of real shocks, flexible inflation targeting outperforms strict inflation targeting while flexible exchange rates outperforms fixed exchange rates. In contrast to Laxton and Pesenti (2003) and Parrado (2004), the present study considers commodity terms of trade shocks and evaluates their implications for monetary policy. It also distinguishes between tradable and non-tradable goods which provides a greater role for relative prices.

In a model featuring traded and non-traded sectors Cashin et al. (2004) analyse the role of commodity terms of trade for real exchange rate dynamics in commodity exporting countries. They derive the commodity terms of trade from the decomposition of the real exchange rate and empirically show that real exchange rate is driven by commodity terms of trade movements in commodity exporting countries. However, the model is not simulated and did evaluate the monetary policy implications of commodity terms of trade shocks. Also, the focus on real exchange rate only left questions about the dynamics of other macroeconomic variables unanswered. The present model is simulated and answer some of these questions.
Santacreu (2005) develops a multi-sector Bayesian DSGE model for New Zealand, a commodity dependent developing economy. She compares the performance of CPI inflation targeting and non-traded inflation targeting reaction functions in the face of shocks. The results show that if the central bank cares more about inflation stabilisation, it should react to CPI inflation, but if it places more weight on output stabilisation, it should react to non-traded inflation. Nevertheless, the model did not take explicit account of the commodity sector.

Gali and Monacelli (2005) build a small open economy model to analyse the properties and macroeconomic implications of CPI inflation targeting, domestic inflation targeting and exchange rate targeting. They show that when the regimes are ranked, domestic inflation targeting yield better stabilisation outcomes than CPI and exchange rate targeting especially with respect to inflation and output gap. However, the performance of domestic inflation targeting comes at the expense of larger volatility in nominal and real exchange rates. In terms of welfare, they show that domestic inflation targeting is more optimal than CPI and exchange rate targeting. The present study differs with Gali and Monacelli (2005) in that it formulates a multi-sector DSGE model with sectorial differences in responses to commodity terms of trade shocks under different monetary policies. It argues that the sectorial structure can significantly affect monetary policy outcomes and welfare.

Similarly, Devereux et al. (2006) develops a small open economy model calibrated to the Asian economies to examine the monetary policy responses to terms of trade and foreign interest rate shocks. They find that CPI inflation targeting is better than non-tradable inflation targeting and exchange rate targeting in stabilising output. However, CPI inflation targeting seems to stabilise the economy at the expense of high exchange rate fluctuations. They also find that financial constraints propagate external shocks but do not alter the ranking of monetary policy rules. In terms of welfare analysis, they show that non-tradable inflation targeting performs better that CPI inflation targeting and exchange rate targeting. However, Devereux et al. (2006) neither incorporate the commodity sector nor consider commodity terms of trade which are the core aspects of the present paper.

In the context of the South Africa, DSGE models are very limited and have been developed to analyse different issues. For example Steinbach et al. (2009), Liu et al. (2009) and Alpanda et al. (2011) develop DSGE models to evaluate business cycle characteristics and forecast the South African economy. Alpanda et al. (2010) also develop a Bayesian DSGE model to analyse the role of the exchange rate in shaping
the South African business cycle. They find that when the exchange rate variable is considered in the model, country risk shocks have sizeable effects on the business cycle and that there is a trade off between inflation and output variability in the short run. Thus, the present DSGE models in South Africa do not evaluate the implications of incorporating the commodity sector in the monetary policy model. This is important because of the role that external shocks play in South Africa’s business cycle.

3 The Model

3.1 Basic outline of the model

A small open commodity exporting economy is considered. This economy has three domestic actors which are; consumers, firms and monetary authorities. There is one external sector which is the rest of the world. There are two production sectors in the domestic economy: traded and non-traded sectors. The traded sector produces primary commodities which are completely exported. This sector is meant to characterise the production and export of commodities such as gold, platinum and diamond in South Africa. The inclusion of the commodity export sector enriches the model and allows the explicit analysis of commodity terms of trade shocks. The non-traded sector produces final goods which are consumed domestically. The domestic traded (commodity export) sector is perfectly competitive while the non-traded goods sector faces monopolistic competition. The external traded sector supplies imports to the domestic economy.

The multi-sector set up is important for South Africa because the economy produces and consumes different products. In fact, the set-up helps to address some questions and debates that cannot be tackled by models with one sector. For example, Santacreu (2005) argues that, the multi-sector framework makes it possible to distinguish between non-traded inflation and CPI inflation. Also, since different sectors are driven by different propagation mechanisms and react differently to external shocks, the separate treatment of these sectors help to design efficient monetary policy rules to respond to external shocks (Batini et al., 2003).

The model also features nominal rigidities in the form of Calvo (1983) staggered price setting in the non-traded sector. This is important because consumer decisions are based on those prices, hence frictions will influence the economy’s response to shocks. Thus it allows the model to reproduce realistic inflation dynamics which include nominal inertia. It also makes the framework to be suitable for the evaluation
of monetary policy (Clarida et al., 1999). Capital stock and investment are assumed to be constant. Consumers own firms and supply labour to the firms and in return they get profits and wages. Labour is assumed to be perfectly mobile across sectors which implies that nominal wages are similar in traded and non-traded sectors. The size of the economy being modelled is small relative to the rest of the world such that the rest of the world can be considered as a closed economy. Monetary policy is modelled as a Taylor rule that incorporates interest rate inertia. The basic structure of the economy is described in Figure 1.

Figure 1: Flow chart of the economy

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\footnote{McCallum and Nelson (1999) argues that the capital stock may be irrelevant for the dynamics of the small open economy because its variation contributes very little to the business cycle fluctuations, at least in the US. Also the inclusion of capital may make the analysis complex because it is a stock while other variables in the model are flows.}
### 3.2 Consumers

There is a representative household who maximises the intertemporal utility subject to an intertemporal budget constraint. The household utility function is:

$$\begin{align*}
U &= E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{C_t^{1-\sigma} L_t^{1+\psi}}{1 - \sigma - \eta \frac{L_t^{1+\psi}}{1+\psi}} \right)
\end{align*}$$

(1)

where; $\beta$ is the subjective discount factor, $\sigma$ is the inverse of the elasticity of substitution between consumption and labour, $\eta$ is the marginal disutility of work and $\psi$ is the inverse of the elasticity of labour supply. $L_t$ is the total labour supply in both traded and non traded sectors. $C_t$ is a composite consumption index composed of non-tradable goods and tradable goods (imports) which takes the constant elasticity of substitution (CES) function of the form:

$$\begin{align*}
C_t &= \left[ \frac{1}{\alpha} C_{Nt}^{\frac{\rho-1}{\rho}} + (1 - \alpha) \frac{1}{\tilde{\psi}} C_{Tt}^{\frac{\rho-1}{\rho}} \right]^{\frac{\frac{\rho}{\rho-1}}{\frac{\rho}{\rho-1}}}
\end{align*}$$

(2)

where $\alpha$ and $1 - \alpha$ are shares of non-tradable and imported goods in total consumption respectively. Implicitly, it is a measure of the degree of openness. $C_{Nt}$ is the consumption of non-traded goods, $C_{Tt}$ is the consumption of imports, $\rho > 0$, is the elasticity of substitution between traded and non-traded goods. Consumption of non-traded goods and imports is differentiated and the elasticity of substitution across varieties is $\lambda$. The consumption indices are represented by the following Dixit and Stiglitz (1977) aggregator:

$$\begin{align*}
C_{Nt} &= \left[ \int_0^1 C_{Nt}(i) \frac{\lambda-1}{\lambda} di \right]^{\frac{1}{\lambda-1}}
\end{align*}$$

(3)

$$\begin{align*}
C_{Tt} &= \left[ \int_0^1 C_{Tt}(i) \frac{\lambda-1}{\lambda} di \right]^{\frac{1}{\lambda-1}}
\end{align*}$$

(4)

where $\lambda > 1$. The consumer’s intertemporal budget constraint is:

$$\begin{align*}
P_t C_t &\leq W_t L_t + \Pi_t + D_t - E_t (Q_{t+1} D_{t+1})
\end{align*}$$

(5)

where $W_t$ is wages, $\Pi_t$ is profits and $D_t$ is the portfolio of assets. $D_{t+1}$ is the nominal payoff of period $t+1$ of the portfolio held at the end of time $t$ and $Q_{t+1}$ is the stochastic discount factor. Minimising expenditure on the total composite demand, the optimal allocations give the following demand functions for non-traded goods and imports.

$$\begin{align*}
C_{Nt} &= \alpha \left( \frac{P_{Nt}}{P_t} \right)^{-\rho} C_t
\end{align*}$$

(6)
\[ C_{Tt} = (1 - \alpha) \left( \frac{P_{Tt}}{P_t} \right)^{-\rho} C_t \]  

(7)

The implied consumer price index is:

\[ P_t = \left( \alpha P_{Nt}^{1-\rho} + (1 - \alpha) P_{Tt}^{1-\rho} \right)^{\frac{1}{1-\rho}} \]  

(8)

where \( P_{Nt} \) and \( P_{Tt} \) are prices of non-traded and import goods respectively. Going forward, \( \rho = 1 \) will be assumed such that the CPI takes the Cobb-Douglas form:

\[ P_t = P_{Nt}^\alpha P_{Tt}^{1-\alpha} \]  

(9)

Thus, the consumer price index is a weighted sum of the prices of traded and non-traded goods. The household optimisation problem gives the following first order conditions:

\[ C_t^\alpha L_t^\psi = \frac{W_t}{P_t} \]  

(10)

\[ \beta R_t E_t \left( \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \left( \frac{P_t}{P_{t+1}} \right) \right) = 1 \]  

(11)

where \( R_t^{-1} \) is the price of a one period bond which is denominated in domestic currency. \( R_t \) is thus the gross interest rate of the bond. Equation (10) is the intratemporal optimal condition which shows the equalisation of marginal utility of consumption to the marginal value of labour. Equation (11) is the consumption Euler equation which represents the trade-off to the economy of moving consumption across time. Log-linearising equation (10) and (11), gives:

\[ \sigma c_t + \psi l_t = w_t - p_t \]  

(12)

\[ c_t = E_t c_{t+1} - \frac{1}{\sigma} (r_t - E_t \pi_{t+1}) \]  

(13)

where small letters denote log deviation from steady state.\(^3\)

### 3.3 Production firms

#### 3.3.1 Domestic Production

There are two sectors in the domestic economy; the traded sector and the non-traded sector. The domestic traded sector produces primary commodities which are all

\(^3\)Note that going forward, we shall use this notation about log deviation from steady state. The log-linearisation is around the steady state.
exported. Firms in the traded sector operate under perfect competition and use the following linear technology:

\[ Y_{Xt} = A_{Xt}L_{Xt} \]  

(14)

where \( A_{Xt} \) is a productivity variable and \( L_{Xt} \) is labour in the commodity export sector. \( A_{Xt} \) follows an AR(1) process such that in logarithms, it is:

\[ \ln A_{Xt} = \rho_X \ln A_{Xt-1} + \epsilon_{Xt} \]  

(15)

where \( \epsilon_{Xt} \sim N(0, 1) \). Cost minimisation in the export commodity sector gives the following marginal cost:

\[ MC^R_{Xt} = \frac{W_t}{A_{Xt}P_{Xt}} \]  

(16)

where \( MC^R_{X} \) is the real marginal cost in the export sector. Log-linearising equation (16) gives:

\[ mc^R_{Xt} = w_t - a_{Xt} - p_{Xt} \]  

(17)

Equation (16) shows the choice of employment which achieve cost minimisation in the commodity export sector.

Firms in the non-traded sector face monopolistic competition and produce differentiated non-traded goods using the linear production technology:

\[ Y_{Nt} = A_{Nt}L_{Nt} \]  

(18)

where \( A_{Nt} \) is a productivity variable and \( L_{Nt} \) is labour in the non-traded sector. \( A_{Nt} \) follows an AR(1) process such that in logarithms, it is:

\[ \ln A_{Nt} = \rho_N \ln A_{Nt-1} + \epsilon_{Nt} \]  

(19)

where \( \epsilon_{Nt} \sim N(0, 1) \). Cost minimisation in the non-traded sector leads to the following optimality condition:

\[ MC^R_{Nt} = \frac{W_t}{A_{Nt}P_{Nt}} \]  

(20)

where \( MC^R_{N} \) is the real marginal cost in the export sector. Log-linearising the marginal cost in the traded sector, gives:

\[ mc^R_{Nt} = w_t - a_{Nt} - p_{Nt} \]  

(21)

As in Cashin et al. (2004), because of perfect competition in the traded sector, the price of tradable goods can be expressed as a function of wages and productivity. Also
the price of non-traded goods can be expressed as a function of the wage, productivity and marginal cost.

\[
P_{Xt} = \frac{W_t}{A_{Xt}}
\]

\[
P_{Nt} = \frac{W_t}{A_{Nt}MC_{Nt}^R}
\]

Since wages are equalised between sectors, the relative price of non-traded goods to tradable goods can be expressed as follows:

\[
P_{Nt} = \frac{A_{Xt}}{A_{Nt}MC_{Nt}^R} P_{Xt}
\]

This shows that the relative price of non-traded goods to primary commodities is determined by technological factors and marginal cost.

### 3.3.2 Foreign Production

Following Cashin et al. (2004), it is assumed that the foreign economy is composed of three production sectors that is non-traded sector, intermediate goods sector, and final goods sector. All foreign production sectors are assumed to operate under perfect competition. Labour is assumed to be mobile across sectors such that wages are equalised across sectors. Production firms in the foreign non-traded goods sector use linear production technologies as follows:

\[
Y_{Nt}^* = A_{Nt}^* L_{Nt}^*
\]

where \( A_{Nt}^* \) is a productivity variable and \( L_{Nt}^* \) is labour in the foreign non-traded sector. \(^4\) \( A_{Nt}^* \) follows an AR(1) process such that in logarithms, it is:

\[
ln A_{Nt}^* = \rho_{Nt} ln A_{Nt-1}^* + \epsilon_{Nt}^*
\]

where \( \epsilon_{Nt}^* \sim N(0,1) \). Production firms in the foreign intermediate goods sector also use the following linear production technology:

\[
Y_{It}^* = A_{It}^* L_{It}^*
\]

where \( A_{It}^* \) is a productivity variable and \( L_{It}^* \) is labour in the foreign intermediate sector. \( A_{It}^* \) follows an AR(1) process such that in logarithms, it is:

\[
ln A_{It}^* = \rho_{It} ln A_{It-1}^* + \epsilon_{It}^*
\]

\(^4\) Going forward, the foreign variables will be indicated by asterisks.
where $\varepsilon_t \sim N(0,1)$. The price of foreign non-traded goods can be expressed as a function of relative productivity and the price of foreign intermediate good as:

$$P_{Nt}^* = \frac{A_{It}^*}{A_{Nt}^*} P_{It}^*$$  \hspace{1cm} (29)

The final good is assumed to be a tradable good and its production uses two intermediate inputs. The first input is the intermediate commodity which is produced in the foreign economy. The second is the commodity which is exported by the domestic economy and other primary commodity producing countries. The traded good is thus produced using the following Cobb-Douglas technology:

$$Y_{Tt}^* = \vartheta(Y_{It}^*)^\nu (Y_{Xt}^*)^{1-\nu}$$

Cost minimisation lead to the following per unit cost;

$$P_{Tt}^* = (P_{It}^*)^\nu P_{Xt}^{1-\nu}$$  \hspace{1cm} (30)

Foreign consumption is assumed to be similar to that in the domestic economy, such that the implied consumer price index is:

$$P_t^* = P_{Nt}^* P_{Tt}^{1-\alpha^*}$$  \hspace{1cm} (31)

### 3.4 Real Exchange rate, Commodity Terms of Trade and Inflation

$$Q_t = \frac{\varepsilon_t P_t}{P_t^*}$$  \hspace{1cm} (32)

The real exchange rate is defined as the domestic price of a basket of consumption relative to foreign price of a basket of consumption. The real exchange rate is decomposed so that it has a commodity terms of trade component. The law of one price is assumed to hold for both exports and imports such that:

$$P_{Xt} = \frac{P_{Xt}^*}{\varepsilon_t}$$  \hspace{1cm} (33)

$$P_{Tt} = \frac{P_{Tt}^*}{\varepsilon_t}$$  \hspace{1cm} (34)

From equation (32), and after some algebra, the following version of the real exchange rate can be derived:\(^5\)

$$Q_t = \left(\frac{A_{Xt}^* A_{Nt}^* P_{Xt}^*}{A_{It}^* A_{Nt}^* P_{It}^*} \right)^\alpha \left( \frac{1}{MC_{Nt}^R} \right)^\alpha$$  \hspace{1cm} (35)

\(^5\)See Appendix B
This is a version of real exchange rate which is similar to Cashin et al. (2004)’s real exchange rate decomposition, where $p_{Xt}^*$ is the commodity terms of trade index, defined as the price of primary commodity with respect to the intermediate foreign good. $\frac{A_{Xt}}{A_{It}}$ shows the productivity differential between the export and import sectors and $\frac{A_{Nt}}{A_{Nt}}$ is the productivity differentials between domestic and foreign non-traded sectors. The two productivity ratios capture the Balassa-Samuelson effect where an increase in productivity in the tradable sector (commodity sector) tends to increase wages in both the tradable and non-tradable sectors and results in increase in price of non-traded goods relative to tradables and an appreciation of the real exchange rate (Cashin et al., 2004).

Commodity terms of trade is defined as:

$$F_t = \frac{p_{Xt}^*}{p_{It}^*}$$  \hspace{1cm} (36)

which can be log linearised to give:

$$f_t = p_{Xt}^* - p_{It}^*$$  \hspace{1cm} (37)

Lagging and taking the difference of equation (37) results in:

$$f_t = f_{t-1} + \pi_{Xt}^* - \pi_{It}^*$$  \hspace{1cm} (38)

Substituting equation (36) into the real exchange rate equation gives:

$$Q_t = \left( \frac{A_{Xt} A_{Nt}^*}{A_{It}^* A_{Nt}^*} F_t \right)^\alpha \left( \frac{1}{MC_{Nt}^R} \right)^\alpha$$  \hspace{1cm} (39)

Log-linearising equation (39), gives:

$$q_t = \alpha(a_{Xt}^* - a_{It}^* + a_{Nt}^* - a_{Nt} + f_t - mc_{Nt})$$  \hspace{1cm} (40)

Foreign traded inflation can be derived from equation (30) by taking the lag and the difference:

$$\pi_{Tt}^* = \nu \pi_{It}^* + (1 - \nu) \pi_{Xt}^*$$  \hspace{1cm} (41)

From equation (9), CPI inflation in the domestic economy can be derived as:

$$\pi_t = \alpha \pi_{Nt} + (1 - \alpha) \pi_{Tt}$$  \hspace{1cm} (42)

From equation (34), imported inflation equation can be derived as:

$$\pi_{Tt} = \pi_{Tt}^* - \Delta e_t$$  \hspace{1cm} (43)

Substituting $\pi_{Tt}^*$ from equation (41) into equation (43), gives:

$$\pi_{Tt} = \nu \pi_{It}^* + (1 - \nu) \pi_{Xt}^* - \Delta e_t$$  \hspace{1cm} (44)
3.5 International Risk Sharing and Uncovered Interest Parity Condition

Complete international markets are assumed where domestic agents have access to foreign securities. This means that the expected nominal return from riskless bonds in the home currency terms is the same as the expected domestic currency return from foreign bonds (Corsetti et al., 2008).\(^6\) This permits the derivation of the international risk sharing condition where consumption risk is perfectly shared between domestic and foreign agents. In fact, the growth of marginal utility of consumption in the same currency units is equalised across countries. Following Gali and Monacelli (2005), the following international risk sharing condition can be derived:

\[
\beta E_t \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \left( \frac{P_t}{P_{t+1}} \right) = \beta E_t \left( \frac{C^*_{t+1}}{C^*_t} \right)^{-\sigma} \left( \frac{\varepsilon_t P^*_t}{\varepsilon_{t+1} P^*_{t+1}} \right) \tag{45}
\]

As shown in Gali and Monacelli (2005) solving equation (45) and iterating result in:

\[
C_t = \Omega Q_t^{-1} C^*_t \tag{46}
\]

where \(\Omega\) is a constant that represents initial asset positions. Log-linearising lead to:

\[
c_t = c^*_t + \frac{1}{\sigma} q_t \tag{47}
\]

Under complete international markets, the uncovered interest parity condition can be derived:

\[
E_t Q_{t+1} (R_t - R^*_t \frac{\varepsilon_{t+1}}{\varepsilon_t}) = 0 \tag{48}
\]

Log-linearising around the steady state gives:

\[
r_t - r^*_t = E_t \Delta e_{t+1} \tag{49}
\]

Equation (49) is the uncovered interest parity condition which relates expected variations of nominal exchange rates to interest rate differentials. Equation (49) can be combined with the real exchange rate equation to give:

\[
E_t \Delta q_{t+1} = (r_t + E_t \pi_{t+1}) - (r^*_t + E_t \pi^*_{t+1}) \tag{50}
\]

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\(^6\)As noted by Parrado (2004), the assumption of complete international markets is motivated by the fact that it eliminates foreign asset movements from the open economy dynamics. This makes steady state unique, where consumption is independent of past shocks.
3.6 Domestic price setting

Non-traded good firms follow Calvo (1983) staggered price setting where they adjust their prices only with some probability. That is at period \( t \), \( 1 - \theta_N \) firms set prices optimally and \( \theta_N \) keep prices unchanged, where \( \theta_N \in (0, 1) \). \( \theta_N \) measures the degree of nominal rigidity. The larger this parameter, the less frequently prices are adjusted, that is prices are more sticky. The general price index at each period evolves according to:

\[
P_{Nt} = \left\{ (1 - \theta_N)P_{Nt}^{new} + \theta_N P_{Nt-1} \right\}^{\frac{1}{1-\varepsilon}}
\]

where \( P_{Nt}^{new} \) is the price level of an optimising firm. The firms that reoptimise their prices at time \( t \) solve the following maximisation problem:

\[
Max \sum_{t=0}^{\infty} (\theta_N)^k E_t \{ Q_{t+k} Y_{t+k} \left( P_{Nt}^{new} - MC_{Nt+k}^n \right) \} \text{ s.t } Y_{t+k} \leq \left( \frac{P_{Nt}^{new}}{P_{Nt+k}} \right)^{-\varepsilon} \left( C_{Nt+k} + C_{Nt+k}^{new} \right)
\]

where \( MC_{Nt+k}^n \) is the nominal marginal cost and \( \theta_{k}^N E_t Q_{t+k} \) is the effective stochastic discount factor. The first order condition for the problem is:

\[
\sum_{t=0}^{\infty} (\theta_N)^k E_t \{ Q_{t+k} Y_{t+k} \left( P_{Nt}^{new} - \frac{\varepsilon}{\varepsilon - 1} MC_{Nt+k}^n \right) \} = 0
\]

Further computations lead to:

\[
\pi_{Nt} = \beta E_t \pi_{Nt+1} + \lambda_{Nt} mc_{Nt}^R
\]

where \( \lambda_{Nt} = \frac{(1-\beta)\theta_N(1-\theta_N)}{\theta_N} \). Equation (54) is the New Keynesian Phillips curve equation for the non-traded sector. It says that inflation is a function of next period’s expected inflation and the real marginal cost.

3.7 Monetary Policy Rule

The model is closed by describing how monetary policy is conducted. Much recent work on monetary policy has modelled monetary policy as an interest rate feedback rule which prescribes how a central bank should adjust policy interest rates in response to economic conditions (see e.g. Clarida et al., 2000; Benigno, 2004). The Taylor (1993) rules have been made prominent in literature. There are many reasons why the Taylor rules are preferable in describing monetary policy. In literature, Taylor rules

\footnote{see Appendix D for the derivation}
have been found to capture well the behavior of monetary policy in many countries (see e.g. Clarida et al., 2000; Lubik and Schorfheide, 2007). Clarida et al. (1999) and Woodford (2003) show that these rules are generally robust and consistent with the main principles of optimal monetary policy. For example in the US, they are consistent with optimal rules derived from baseline and hybrid models. They are also flexible in nesting a wide range of alternative monetary policy strategies. More importantly, Taylor rules have been found to provide determinacy (Clarida et al., 1999). This means that these rules ensure a unique stationary rational expectations equilibrium in the model. In these models, the asymptotic response of policy rate with respect to inflation should be greater than one for these models to be stable. Following Ortiz and Sturzenegger (2007), Lubik and Schorfheide (2007) and Alpanda et al. (2010) this study assumes the following generalised Taylor rule:

\[
R_t = R^\theta_{t-1} \left\{ \left( \frac{Y_t}{\pi_t} \right)^{\omega_1} \left( \frac{\pi_t}{\bar{\pi}} \right)^{\omega_2} \left( \frac{\pi_{Nt}}{\bar{\pi}_N} \right)^{\omega_3} \left( \frac{\varepsilon_t/\bar{\varepsilon}_{t-1}}{\bar{\pi}} \right)^{\omega_4} \right\}^{1-\rho_r} (55)
\]

The log-linearised version of the monetary policy rule is:

\[
r_t = \rho_r r_{t-1} + (1 - \rho_r) (\omega_1 y_t + \omega_2 \pi_t + \omega_3 \pi_{Nt} + \omega_4 \Delta \varepsilon_t) + \varepsilon_{r,t} (56)
\]

where \(\omega_1, \omega_2, \omega_3, \text{and} \omega_4\) allows the monetary authority to control the output, CPI inflation, non traded inflation and nominal exchange rate respectively. To allow for the comparison of different monetary policy regimes, the parameters are changed so that one monetary policy regime can be specified at a time.\(^8\) \(\rho_r\) is the smoothing parameter. The smoothing parameter is included in this specification for several reasons. For instance, several empirical studies have observed that central banks change interest rates in sequences of small steps in the same direction. Indeed, Clarida et al. (2000) argue that policy reaction functions without interest rate smoothing are too restrictive to describe the actual interest rate changes in most central banks. Also, Sack and Wieland (2000), argue that the presence of uncertainty about the relevant model parameters, the structure of the economy and concerns about the soundness of the financial system may make central banks to have interest rate smoothing. This helps to avoid unintended fluctuations in economic activity and adverse reactions of financial markets to frequent changes in interest rates.

\(^8\)For CPI targeting rule, it is considered that \(\omega_3 = \omega_4 = 0\), for non-traded inflation targeting rule \(\omega_2 = \omega_4 = 0\) and for exchange rate targeting rule, \(\omega_2 = \omega_3 = 0\).
3.8 Equilibrium

In equilibrium, the markets for non-traded goods, traded goods and labour must clear. The goods market clearing condition in the domestic economy requires that total domestic production made up of non-traded output and exported output is equal to total demand. That is:

\[ Y_t = Y_{Nt} + Y_{Xt} \]  

(57)

where \( Y_{Nt} = C_{Nt} \) and \( Y_{Xt} = C_{Xt} \). Log log-linearising (57):

\[ y_t = y_{Nt}(Y_{Nt}) + y_{Xt}(Y_{Xt}) \]  

(58)

Using \( Y_{Nt} = C_{Nt} \) and combining with equation (6) results in:

\[ y_{Nt} = -\rho(1 - \alpha) [p_{Nt} - e_t - p_{Mt}^*] + c_t \]  

(59)

Equation (59) is the equilibrium condition for the non-traded sector. The equilibrium condition for the commodity export sector is given by:

\[ Y_{Xt} = Y_{Xt}^* = C_{Xt}^* \]  

(60)

Using the equation for the consumption of exports, it can be shown that:

\[ Y_X = \left(\frac{1 - \nu}{\nu}\right)^\nu Y_{Tt}^* \left(\frac{P_{Xt}^*}{P_{It}^*}\right)^\nu \]  

(61)

The log-linearised version of (61) is:

\[ y_{Xt} = y_{Tt}^* + \nu(p_{Xt}^* - p_{It}^*) \]  

(62)

Thus, the equilibrium condition depicting the IS equation for the domestic economy is given by:

\[ y_t = \left(\frac{Y_{Nt}}{Y}\right)(-\rho(1 - \alpha) [p_{Nt} - e_t - p_{Mt}^*] + c_t) + \left(\frac{Y_{Xt}}{Y}\right)(y_{Tt}^* + \nu(p_{Xt}^* - p_{It}^*)) \]  

(63)

where \( \frac{Y_{Nt}}{Y} \) and \( \frac{Y_{Xt}}{Y} \) are steady state ratios of the non-traded goods and exports to total income.

The supply side of the model is given by the marginal cost equations in both the commodity export sector and the non-traded sectors. For the commodity export sector marginal cost is given by:

\[ mc_X^R = w_t - p_X - a_X \]  

(64)
Combining equation (64) with (12), (9) and (34) gives the new expression for the marginal cost in the commodity export sector:

$$mc^R_X = \sigma c_t + \psi l_t + (1 - \alpha)(p^*_T - e_t) + \alpha p_{Nt} - p^*_X + e_t - a_X$$  \hspace{1cm} (65)$$

In the non-traded sector, the marginal cost is given by:

$$mc^R_{Nt} = w_t - p_{Nt} - a_{Nt}$$ \hspace{1cm} (66)$$

Combining equation (66) with (12), (9) and (34) gives the final marginal cost function:

$$mc^R_X = \sigma c_t + \psi l_t + \alpha p_{Nt} + (1 - \alpha)(p^*_T - e_t) - p_{Nt} - a_{Nt}$$ \hspace{1cm} (67)$$

The labour market must clear. The market clearing condition is:

$$L_t = L_{Xt} + L_{Nt}$$ \hspace{1cm} (68)$$

Log-linearising equation (68) and substituting equations (18) and (14) into equation (68) lead to:

$$l_t = \frac{L_X}{L}(y_{Xt} - a_{Xt}) + \frac{L_N}{L}(y_{Nt} - a_{Nt})$$ \hspace{1cm} (69)$$

Finally, the equilibrium conditions in the foreign economy implies that:

$$y^*_t = \alpha^* y^*_{Nt} + (1 - \alpha^*) y^*_{Tt}$$ \hspace{1cm} (70)$$

$$y^*_{Nt} = -\rho^* p^*_{Nt} + \rho^*(\alpha^* p^*_{Nt} + (1 - \alpha^*) p^*_T) + y^*_t$$ \hspace{1cm} (71)$$

$$y^*_{Tt} = \nu y^*_t + (1 - \nu) y^*_{Xt}$$ \hspace{1cm} (72)$$

$$r^*_t = \rho^* r^*_{t-1} + \epsilon^*_{r,t}$$ \hspace{1cm} (73)$$

The general equilibrium is characterised by a sequence of \(y_t, y_{nt}, y_{Xt}, c_t, r_t, \pi_t,

\pi_{Nt}, \pi_{Tt}, a_{nt}, a_{Xt}, p_{Nt}, e_t, q_t, mc_{Nt}, mc_{Xt}, f_t, l_t, y^*_t, y^*_{Nt}, y^*_t, y^*_{Tt}, r^*_t, \pi^*_{Tt},

\pi^*_{It}, x^*_{Xt}, p^*_{Nt}, p^*_{Tt}, p^*_{It}, a^*_t, a^*_{Nt} \) that gives the solution of the equations for the domestic and foreign economies.

### 4 Calibration and Solution

In order to solve the model, the values of parameters need to be determined. The model is calibrated to match the key features of the South African economy. Parameters are obtained from the South African data from 1990-2008, previous studies based
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>Elasticity of substitution between traded and non-traded goods</td>
<td>1</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Share of non-traded goods in consumption</td>
<td>0.8</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Subjective discount factor</td>
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</tr>
<tr>
<td>$\sigma$</td>
<td>Inverse of the elasticity of substitution between consumption and labour</td>
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</tr>
<tr>
<td>$\psi$</td>
<td>Inverse of the elasticity of labour supply</td>
<td>6</td>
</tr>
<tr>
<td>$\theta_N$</td>
<td>Stickiness parameter in the non traded sector</td>
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</tr>
<tr>
<td>$\rho_{\pi*}$</td>
<td>Persistence parameter for foreign inflation</td>
<td>0.5</td>
</tr>
<tr>
<td>$\rho_{r*}$</td>
<td>Persistence parameter for foreign interest rate</td>
<td>0.8</td>
</tr>
<tr>
<td>$\rho_{\tau*}$</td>
<td>Smoothing parameter for Taylor rule</td>
<td>0.73</td>
</tr>
<tr>
<td>$\rho_{\kappa*}$</td>
<td>Persistence parameter of labour productivity in the non traded sector</td>
<td>0.743</td>
</tr>
<tr>
<td>$\rho_{x*}$</td>
<td>Persistence parameter of labour productivity in the commodity export sector</td>
<td>0.85</td>
</tr>
<tr>
<td>$\rho_{r*}$</td>
<td>Persistence parameter of labour productivity in foreign intermediate sector</td>
<td>0.8</td>
</tr>
<tr>
<td>$\rho_{n*}$</td>
<td>Persistence parameter of labour productivity in foreign non-traded sector</td>
<td>0.8</td>
</tr>
<tr>
<td>$\alpha^*$</td>
<td>Share of non-traded goods in the foreign country</td>
<td>0.8</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Share of exported commodity (by domestic economy) in foreign production</td>
<td>0.26</td>
</tr>
<tr>
<td>$\rho_{p_x^*}$</td>
<td>Persistence parameter of world price of export commodity</td>
<td>0.8</td>
</tr>
<tr>
<td>$\rho_{p_I^*}$</td>
<td>Persistence parameter of foreign price of intermediate good</td>
<td>0.8</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Elasticity of substitution across varieties</td>
<td>10</td>
</tr>
<tr>
<td>$\omega_1$</td>
<td>Weight on output in the Taylor rule</td>
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</tr>
<tr>
<td>$\omega_2$</td>
<td>Weight on CPI inflation in the Taylor rule</td>
<td>1.5</td>
</tr>
<tr>
<td>$\omega_3$</td>
<td>Weight on domestic inflation in the Taylor rule</td>
<td>1.5</td>
</tr>
<tr>
<td>$\omega_4$</td>
<td>Weight on exchange rate in the Taylor rule</td>
<td>0.05</td>
</tr>
</tbody>
</table>
on the South African economy and business cycle literature on small open economies. The benchmark parameters are described in Table 1.

The elasticity of substitution between traded goods and non-traded goods is set at 1 following Devereux et al. (2006) and Santacreu (2005). In line with real business cycles literature, the discount factor is set at 0.99 implying that the steady state real interest rate is about 4% per annum. Following Alpanda et al. (2010), the inverse of the elasticity of labour supply is assumed to be 6 which reflects that in steady state, the gross wage mark-up is about 1.2 over the marginal rate of substitution. This estimate lies in the plausible range of 1.2 and 1.7 suggested by literature on small open economies. Steinbach et al. (2009) estimates the value of the elasticity of substitution between consumption and labour for the South African economy to be 1 using Bayesian technique. This model adopts this estimated parameter.

The import share in consumption is assumed to be 0.2, consistent with estimates by Steinbach et al. (2009) for South Africa for the period 2002-2007. They find that the import penetration ratio to total GDP is about 30% and the import penetration in consumption is about is 7.5% during this period. Using this fact, it implies that the share of non-traded goods in consumption is about 0.8. However, these parameter values are changed through experiments in sensitivity analysis.

The autoregressive productivity parameter in the non-traded goods sector is set at 0.743 based on estimates by Alpanda et al. (2010). Following Santacreu (2005), the productivity parameter in the commodity export sector is set at 0.85 while the foreign productivity parameter in the non-traded goods production and the intermediate goods production are both set at 0.8.

As in Alpanda et al. (2010) and Steinbach et al. (2009), the degree of nominal price rigidity is set at 0.75, which suggest that prices are adjusted on average after 4 quarters in South Africa. This allows the model to generate sensible impulse responses. The weight on CPI inflation and non-traded inflation in the Taylor rule, $\omega_2$ and $\omega_3$ are both set initially at 1.5 while the weight on output, $\omega_1$ is set at 0.5 following Steinbach et al. (2009). However these weights are varied in order to evaluate the implications their variations for welfare. The weight on the exchange rate $\omega_4$ is initially set at 0.05. Following Ortiz and Sturzenegger (2007) the interest rate smoothing parameter is calibrated at 0.73. This setting allows the expectations to play a role in shaping future economic developments in South Africa. It is believed that this value best characterise the recent South African Reserve Bank’s efforts to reduce interest rate volatility and make monetary policy to be more predictable. The share of non-
traded goods in the foreign consumption is assumed to be 0.8. This implies that the share of traded goods is about 0.2. Most steady state parameters are obtained from equilibrium relations in the model.

The model is solved using Dynare programme which uses the Blanchard and Kahn (1980) algorithm to solve the linearized model (see Juillard, 1996). The analysis focuses mainly on commodity terms of trade shocks but later considers productivity shocks in the commodity export sector.

5 Results

5.1 Impulse response analysis

This section describes the dynamic responses of selected macroeconomic variables to commodity terms of trade shocks and productivity shocks in the export sector under alternative monetary policy rules.

5.1.1 Commodity terms of trade shocks

Figure 2 presents impulse responses to a one standard deviation positive shock to commodity terms of trade. The figure show that the positive commodity terms of trade shocks results in higher output in the export sector. Output in the non-traded sector falls. Aggregate output also falls because of the bigger impact from the non-traded sector. The fall in the output of the non-traded sector could be attributed to the substitution effect in production which entails the movement of resources to the booming commodity export sector. As expected, the increase in exports induces appreciation of the nominal and real exchange rates. The real appreciation generates a wealth effect which increases demand for non-tradable goods. The increase in demand for non-tradable goods results in overheating of the economy which puts pressure for non-traded good prices to increase. The increase in prices of non-traded goods results in increase in CPI inflation. Central banks respond to rising inflation by raising interest rates. Following monetary contraction and reduction in output, aggregate consumption decreases. This may be a result of monetary contraction which makes consumption more expensive.

9The DYNARE programme can derive the reduced-form representation of the model and then provides standard moments based on assumptions about the stochastic processes. Blanchard and Kahn (1980) show that if the number of eigen values outside the unit circle is equal to the number of non-predetermined variables, then there exists a unique rational expectations solution to the system.
Figure 2: Impulse responses to commodity terms of trade shock
The response patterns of most variables depends on the monetary policy regimes. For aggregate output, the responses are relatively similar in all monetary policy regimes. Non-traded output falls on impact while export output increases, possibly reflecting the symptoms of the Dutch disease.\textsuperscript{10} The responses of sectorial (traded and non-traded) output and consumption are higher under NTIT and smallest under CIT.

The CIT rule also experiences a larger appreciation of the nominal exchange rate than under NTIT and ET. As pointed out by Obstfeld and Rogoff (1996) this is intuitive because under flexible exchange rate system, the presence of sticky prices makes the adjustment to terms of trade shocks to take place through changes in the nominal exchange rates. As expected, the commodity terms of trade shock generates an initial appreciation of real exchange rate in all regimes. Possibly this reflects the "commodity currency" effect. This was also highlighted by Cashin et al. (2004) who find strong evidence of a long-run relationship between real exchange rate and commodity terms of trade movements for commodity exporting countries. The impact of the shock on the real exchange rate is much larger under CIT than under NTIT and ET. The possible explanation is that the presence of the floating exchange rates under CIT regime implies active use of the exchange rate channel to stabilise other variables like CPI inflation (Svensson, 2000). Under ET, real exchange rate is less volatile due to stable nominal exchange rate. According to Mussa (1986), this stability reflects excess smoothness of the exchange rate. Devereux et al. (2006), Parrado (2004), and Gali and Monacelli (2005) find similar results where larger real exchange rate and nominal exchange rate responses are observed under CIT and NTIT regimes than in exchange rate pegs.

The response pattern of non-traded prices which shows initial rise and then decrease over time is generally similar in all regimes. According to Chung et al. (2007), when prices are sticky, a positive real shock decreases marginal costs which, despite an initial rise, progressively reduce the non-traded prices. However, the initial response of non-traded inflation is larger under NTIT but moderate and persistent under CIT. Also, the initial response of CPI inflation is larger under NTIT and ET but more muted under CIT. The low CPI inflation response under CIT rule can be explained by the presence of flexible exchange rates which reduce the direct effects of commod-

\textsuperscript{10}The Dutch disease exists when the boom in one sector (traded sector) induces appreciation of the real exchange rate which reduces the competitiveness of the non-booming sector (non-traded sector)
ity terms of trade shocks on inflation. Another possible explanation is that inflation expectations are well anchored under CIT.

In all cases, central banks respond to the rise in inflation by increasing interest rates. The NTIT rule raises interest rates more sharply to accommodate the shock while the response under CIT is moderate. ET central banks display very small and less persistent interest rate responses. Under CIT, the moderate response can be explained by the presence of interest rate smoothing aimed at making monetary policy more predictable and credible. The small interest rate responses under ET can possibly imply that ET central banks do not use interest rates, but instead use reserves as instruments of monetary policy (Benes et al., 2008).

Overall, the dynamic adjustment of most variables shows that CIT rule results in generally small responses, followed by ET and NTIT. However, the stabilisation of most variables under CIT comes at a cost of increased exchange rate fluctuations.

5.1.2 Export productivity shock

Although the hallmark of the study is to evaluate the responses to commodity terms of trade shocks, the introduction of the commodity sector in the model makes the analysis of responses to productivity shocks in the commodity export sector interesting. Since the commodity is all exported, the productivity shock is closely related to the terms of trade shock. It also helps to examine the presence of the Balassa-Samuelson effect. Figure 3 presents impulse responses of variables to an export sector productivity shock.

The export sector productivity shock increases aggregate output and exported output while non-traded output decreases. In the presence of the two opposing effects on output from the traded and non-traded sectors, the expansionary effect of the former seems to be larger than the contractionary effect of the latter. The expanding export sector generates nominal and real exchange rate appreciations. However, the real exchange rate may also appreciate due to increase in interest rates. Because of labour mobility across sectors, wages are equalised, thus the increase in productivity in the traded sector raises wages also in the non-traded sector. As a result, marginal costs are pushed up, resulting in higher prices of non-traded goods. This results in rising inflation especially under ET and CIT. Central banks respond by raising interest rates. The dynamic responses of these variables suggest the presence of the Balassa-Samuelson effect where an increase in productivity in the traded sector appreciates the real exchange rate and increases prices of non-tradable goods through
Figure 3: Impulse responses to an export sector productivity shock
wage equalisations (see e.g. Obstfeld and Rogoff, 1996).

The comparison of responses under different monetary policy regimes show that the adjustment patterns of aggregate output are relatively similar in all regimes, but non-traded output and exported output fluctuate more under NTIT than in other regimes. This suggests that non-traded output is very sensitive to price changes generated by productivity shocks in the export sector. Consumption decreases on impact following the productivity shocks, with largest decline being observed under NTIT rule. The fall in consumption may be explained by the substitution effects between traded and non-traded goods which are stronger than the income effects. The largest fall in consumption under NTIT is not a surprise given that there is a large proportion of non-traded goods in the consumption basket.

The nominal and real exchange rates are more volatile under CIT than in other regimes. According to Obstfeld and Rogoff (2000), most NOEM models, have stabilisation trade-offs such that optimal monetary policy can involve some exchange rate fluctuations following productivity shocks. This suggests that the CIT rule stabilise output using the exchange rates. This contrasts the findings of Gali and Monacelli (2005) who observe a more muted depreciation of the real exchange rate following a domestic productivity shock.

Under ET and CIT, the rise in non-traded good prices generates higher non-traded inflation and CPI inflation, but under NTIT, this is not the case. The responses of both non-traded inflation and CPI inflation under NTIT are more muted, both on impact and along the transition. Intuitively, NTIT rule is more active when responding to shocks, as reflected by its large interest rate responses. The general picture which seems to emerge from the proceeding analysis is that CIT stabilise most of the variables. However, the stabilisation of these variables involve substantial movements in real and nominal exchange rate movements.

5.2 Volatility and welfare evaluations of alternative monetary policies

5.2.1 Volatility analysis

Table 2 shows volatilities of selected macroeconomic variables. The results show that total output and non-traded output exhibits lowest volatility under CIT and NTIT and highest under ET. The intuition is that in the presence of sticky prices at least in some sectors of the economy, adjustments under exchange rate targeting regimes
Table 2: Volatility analysis

<table>
<thead>
<tr>
<th></th>
<th>CIT</th>
<th>NTIT</th>
<th>ET</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>0.06</td>
<td>0.07</td>
<td>0.07</td>
</tr>
<tr>
<td>Non-traded output</td>
<td>0.02</td>
<td>0.02</td>
<td>0.03</td>
</tr>
<tr>
<td>Exported output</td>
<td>0.25</td>
<td>0.39</td>
<td>0.28</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.09</td>
<td>0.41</td>
<td>0.33</td>
</tr>
<tr>
<td>Nominal exchange rate</td>
<td>0.33</td>
<td>0.24</td>
<td>0.01</td>
</tr>
<tr>
<td>Real exchange rate</td>
<td>0.22</td>
<td>0.11</td>
<td>0.10</td>
</tr>
<tr>
<td>Non-traded prices</td>
<td>0.08</td>
<td>0.05</td>
<td>0.08</td>
</tr>
<tr>
<td>Non-traded inflation</td>
<td>0.19</td>
<td>0.09</td>
<td>0.25</td>
</tr>
<tr>
<td>CPI inflation</td>
<td>0.52</td>
<td>1.10</td>
<td>2.36</td>
</tr>
<tr>
<td>Interest rates</td>
<td>1.44</td>
<td>0.03</td>
<td>0.03</td>
</tr>
</tbody>
</table>

CIT is CPI inflation targeting, NTIT is non-traded inflation targeting and ET is exchange rate targeting. The numerical values are standard deviations.

entails higher volatility in the real sector. Exported output and consumption are less volatile under CIT. This is consistent with Santacreu (2005) and Devereux et al. (2006) who also conclude that responding to CPI inflation generates less volatility in output and consumption. By design, ET delivers substantially lower volatility of both nominal exchange rate and real exchange rate, while CIT involve higher volatility of these variables. This result is a confirmation of earlier impulse responses where the exchange rates display larger responses under CIT. The intuition is that the process of stabilising output and consumption by CIT rule involve substantial movements in the exchange rates. NTIT generates lowest volatility of non-traded prices which results in lower volatility of non-traded inflation. This shows that the NTIT policy is effective in stabilising non-traded inflation. As expected, CIT delivers low volatility in CPI inflation, but substantially higher fluctuations in interest rates than under NTIT and ET.

5.2.2 Welfare implications of alternative monetary policy regimes

The welfare implications of alternative monetary policy rules in a commodity dependent economy is also analysed. Much of the literature on optimal monetary policy assumes that the central bank aims at minimising the deviations of output and inflation from target values using loss functions (see e.g. Clarida et al., 1999; Laxton and Pesenti, 2003; Alpanda et al., 2010). The present study follows these approaches and consider a loss function which translate the behavior of policy targets into some
aggregate welfare measure. Thus, the objective of the central bank is to minimise welfare losses from deviations of output, inflation, interest rates and exchange rates from their steady state values.

The loss function is appealing as a welfare measure for several reasons. Firstly, it allows the incorporation of some aspects of interest-rate smoothing. This is consistent with the principle of optimal monetary policy since interest rate smoothing is introduced to capture policy inertia (Clarida et al., 1999). Secondly, as noted by Adolfson et al. (2011), loss functions can be formulated in terms of observable macroeconomic variables for example inflation and output gap. This provides a better guide to welfare analysis especially in economies which are characterised by massive economic fluctuations. Thirdly, Clarida et al. (1999) argue that loss functions capture the major cost of inflation that is uncertainty generated from inflation variability. Fourthly, loss functions provide a reasonable optimising framework which simplifies welfare estimations (see e.g Woodford, 2003). Thus they allow simple characterisation of central bank preferences in line with the way monetary policy operates in practice.

The welfare loss function considered is:

\[ L_t = \lambda_\pi \pi_t^2 + \lambda_y y_t^2 + \lambda_e e_t^2 + \lambda_r r_t^2 \]  

Taking unconditional expectations, the loss function can be expressed as:

\[ E(L_t) = \lambda_\pi Var(\pi_t) + \lambda_y Var(y_t) + \lambda_e Var(e_t) + \lambda_r Var(r_t) \]  

where \( \pi_t \) is a measure of inflation depending on the choice of inflation under consideration (CPI or non-traded inflation), \( y_t \) is output, \( e_t \) is nominal exchange rates, \( r_t \) is the interest rate. \( Var(\pi_t), Var(y_t), Var(e_t) \) and \( Var(r_t) \) are the unconditional variances of inflation, output, nominal exchange rates and interest rates. The policy rule coefficients are optimally chosen to minimize a quadratic loss function. Lower values of

---

11The alternative is to estimate welfare using utility function of the representative consumer. However, a limitation of this approach is that it does not involve any interest-rate smoothing, a feature that has been found to be necessary especially in comparing the performance of different monetary policy rules.

12Woodford (2003) shows that it is possible to motivate a quadratic loss function as a second order Taylor series approximation of the expected utility of the economy’s representative household which is equal to the expected discounted sum of period losses for certain coefficients. He also shows that a linear approximation to the policy function is sufficient to accurately approximate welfare up to a second order if the second order approximation to the welfare function contain quadratic terms. The welfare loss is proportional to the expected discounted sum of squared deviations of variables.

13The optimal parameters are computed using Dynare’s optimal simple rule algorithm. In this computation, Dynare searches numerically the parameters of the policy function that minimises the weighted variance of variables.
Table 3: Welfare losses of alternative monetary policy regimes

<table>
<thead>
<tr>
<th>Weights</th>
<th>Welfare losses</th>
</tr>
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<tbody>
<tr>
<td>(\lambda_\pi)</td>
<td>(\lambda_y)</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>1</td>
<td>0.5</td>
</tr>
<tr>
<td>1.5</td>
<td>0.5</td>
</tr>
<tr>
<td>2</td>
<td>0.5</td>
</tr>
<tr>
<td>0.5</td>
<td>1</td>
</tr>
<tr>
<td>0.5</td>
<td>1.5</td>
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<tr>
<td>0.5</td>
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</table>

CIT is consumer price index inflation targeting, NTIT is non tradable inflation targeting and ET is exchange rate targeting.

the loss function shows less welfare losses. Thus the optimal monetary policy is the one that minimises the loss function (74). Alternative values of loss function weights reflecting different central bank preferences in the range 0.5 to 2 are considered.

Table 3 reports the results of the welfare losses of different monetary policy rules. The weights on each variable in the loss function reflects central banks preference in terms of stabilising those variables. The results show that when the central bank places equal weight of 0.5 on all the variables in the loss function, CIT achieves the lowest welfare losses followed by NTIT and lastly ET. When the weight on inflation is increased, while the weights on other variables are kept constant, CIT delivers less welfare losses. In general the ranking is CIT, NTIT and ET. Using utility based welfare measures, Devereux et al. (2006) finds a different ranking. He ranks NTIT first, followed by CPI targeting and lastly ET in a model with financial constraints and complete exchange rate pass-throughs. The differences may be attributed to the differences in welfare in the models.

The central bank preferences are also varied in terms of output stabilisation while keeping the weights on other variables constant. In that case, NTIT delivers lowest welfare losses while large welfare losses are observed under ET. Intuitively, targeting non-traded inflation avoids excessive volatility in interest rates which subsequently
reduces volatility in output and welfare losses. The last result is consistent with the findings of Chia and Alba (2006) who observe that exchange rate pegs achieve less welfare compared to flexible exchange rates because they do not stabilise output. The ranking of the regimes in this case implies that NTIT has lowest welfare losses, followed by CIT and lastly, ET. This result is consistent with Santacreu (2005) and Svensson (2000) who conclude that the central bank which is concerned about output stabilisation should target non-traded inflation. The intuition is that when the central bank responds to overall CPI inflation, it attempts to offset the direct effects of exchange rate movements which are largely temporary. On the other hand, responding to non-traded inflation implies that the central bank ignores the direct exchange rate impact on CPI but focuses on the direct effect through the output gap (Santacreu, 2005).

When more weight is put on interest rates, CIT delivers lowest welfare loss, followed by NTIT and lastly ET. According to Parrado (2004), welfare losses tend to be higher for fixed exchange rate regimes than for flexible exchange rate regimes if shocks are real because fixed exchange rates do not allow smooth adjustment of macroeconomic variables to shocks. As the central bank becomes more concerned about exchange rate stabilisation, ET regime dominates in terms of delivering less welfare losses than other regimes. By design, this regime reduces exchange rate fluctuations which help to generate less welfare losses.

6 Sensitivity Analysis

This section investigates the sensitivity of the reported results to changes in openness, price stickiness and the elasticity of substitution between traded goods and non-traded goods.

6.1 Sensitivity tests on impulse response functions

6.1.1 Openness

Trade openness has increased in emerging market economies in recent years, reflecting higher investment in the traded good sectors, structural reforms and the intensive use of imported intermediate inputs in the production final goods (Laxton and Pesenti, 2003). It is therefore imperative to evaluate if changes in openness affect the results. Figure 4 present the impulse response functions when the degree of openness is increased from 0.2 to 0.6. The response patterns of non-traded output, consumption,
real exchange rate and non-traded good prices are generally similar to the benchmark calibration case. However in terms of sizes, they are marginally larger than in the benchmark, suggesting that the impact of commodity terms of trade shocks on the economy increases with openness. This is expected considering that openness increases demand for foreign goods, and thus may reduce demand for domestic goods. Openness also increases the volatility and persistence of nominal and real exchange rates under CIT. This is intuitive considering that CIT regime has flexible exchange rates which tend to fluctuate more to offset volatility in other variables. Also, consistent with the sticky price models, an increase in openness generates volatile exchange rates since they become the adjustment mechanism for the economy.

Higher openness increases the impact of commodity terms of trade shocks on non-traded inflation, CPI inflation especially under CIT. This can be attributed to greater exposure of the economy to external shocks which makes CIT to be more vulnerable. Indeed, CIT relies more on the exchange rate fluctuations. The central banks respond by raising interest rates and the responses are larger under CIT than under ET and NTIT.

6.1.2 Price stickiness

The sensitivity of the results to the degree of stickiness considers a special case where there is no stickiness in the prices of non-traded goods. This case is similar to Cashin et al. (2004)’s framework. Figure 5 shows the impulse responses when there is no stickiness. In this case, non-traded output, exported output and exchange rate responses are generally similar to the baseline calibration where stickiness is high. On the other hand, the response of aggregate output is now higher under NTIT reflecting that when the prices in the non-traded sector are flexible, the effects of the external shock on output is high. The other pattern which emerges from the results is that non-tradable good prices are now more volatile and they adjust fast. This is consistent with sticky price models which suggests that price stickiness acts as a constraint to firms in the non-traded sector and when the constraint is removed, non-tradable prices becomes more volatile. The patterns of responses of nominal and real exchange rate and nominal exchange rate exhibit similar patterns to the baseline case, consistent with the findings of Cashin et al. (2004). Chia and Alba (2006) also conclude that nominal exchange rate depreciation is larger under flexible exchange rates than under pegs because under sticky prices, nominal exchange rates are used to insulate the economy from real shocks.
However, CPI inflation rises sharply and is more persistent under and NTIT, following the rise in non-traded good prices. Under CIT rule, CPI inflation remain relatively stable which suggest that the CIT central bank pays more attention to the mechanism of price adjustment. If prices are flexible, central banks are more aggressive in ensuring that price expectations remain anchored. As a result, they respond by raising interest rates by more basis points under NTIT, than in other regimes when prices are less sticky. This suggests that NTIT central banks consider more flexible prices as a source of inflation.

6.1.3 Elasticity of substitution

Figure 6 shows impulse responses of variables to commodity terms of trade shocks when the elasticity of substitution between traded and non traded goods is low ($\rho = 0.4$). Aggregate output increase and is more persistent following commodity terms of trade shocks. The reason could be that economic agents cannot easily substitute the more expensive domestic goods for less expensive foreign goods, which makes higher income effect to result in increase in demand for non-traded goods. The dynamic adjustment of exported output, nominal exchange rates, real exchange rates, non-traded good prices and interest rates are generally similar to the baseline calibration. Although the response patterns of these variables are similar with the baseline, the responses in the new parameterisation are smaller than in the latter, with the exception of exchange rate responses. Intuitively, higher elasticity of substitution implies greater responsiveness of variables to international relative prices (Gali and Monacelli 2005).

6.2 Sensitivity tests on volatility and welfare evaluations

The performance of the alternative monetary policy rules is also analysed when parameter values of openness, stickiness and elasticity of substitution of traded and non-traded goods are changed. Table 4 in the Appendix display the results. Like in the baseline case, when the degree of openness is high, responding to non-traded inflation generates higher volatility in total output and non-traded output while responding to CIT results in higher nominal, real exchange rate and interest rate volatility. This shows that higher openness increases the sensitivity of output to shocks. On the other hand, non-traded prices, non-traded inflation, CPI inflation and consumption are more volatile under ET. Also, when the value of the stickiness parameter is reduced, ET generates in aggregate output more volatile aggregate output, exported
output, consumption, non-traded prices and CPI inflation. The exchange rate variables and interest rates still display higher volatility under CIT suggesting that even when all sectorial prices are flexible, exchange rates continue to fluctuate more. This pattern is still maintained when the elasticity of substitution between traded goods is low.

Table 5 shows the effects of changing the degrees of openness, stickiness and elasticity of substitution on welfare. When the weights on all variables are the same, higher openness results in less welfare losses under NTIT. However when more weight is put on inflation, openness improves the welfare effects of CIT. Also as in the benchmark calibration case, increasing the weight on output stabilisation as openness increases yields less welfare losses under NTIT, followed by CIT and then ET. It is also apparent from the table that when the economy is more open, an increase in preference towards interest rate smoothing and exchange rate stabilisation is welfare enhancing under the CIT rule. The intuition is that, as the economy becomes more open, the reduction in interest rate and exchange rate fluctuations reduces excess volatility in economic activity in the face of external shocks which in turn reduce welfare losses. When the sticky price assumption is relaxed, CIT performs better than other regimes especially when more weight is put on inflation and interest rate stabilisation. However, more preference on output and exchange rate stabilisation generates less welfare losses under NTIT. On the other hand, reducing the elasticity of substitution between traded goods and non-traded goods under varying degrees of central bank preferences on inflation, output and interest rate stabilisation results in CIT performing better than NTIT and ET. The intuition is that, low elasticity of substitution generates less volatility in the exchange rates which enhances welfare under CIT.

7 Conclusion

The study uses a multi-sector New Keynesian DSGE model to examine the appropriate monetary policy responses to commodity shocks. Particularly, the study evaluates if CPI inflation targeting performs better than NTIT and ET regimes. It also evaluates the optimal monetary policy implications of these shocks using the central bank loss function. The DSGE model features the commodity sector and is characterised by sticky prices and monopolistic competition in the non-traded sector. The model is calibrated to the South Africa economy. The advantage of the framework is that it
allows for microeconomic foundations of the optimising behavior of economic agents.

The analysis shows that in the face of commodity terms of trade shocks, CIT stabilises most variables such as output, consumption, CPI inflation, non-traded inflation and non-traded good prices. However, the stabilisation is at the cost of high real and nominal exchange rate volatility. Even in the face of productivity shocks in the commodity export sector, CIT still exhibit higher volatility of the exchange rates.

The results also show that CIT delivers low volatility of macroeconomic variables but involve substantial volatility in exchange rates. The analysis of welfare show that the central bank achieves less welfare losses when preferences shift towards more inflation stabilisation under CPI inflation targeting than in other regimes. However, if the central bank cares about output stabilisation, it should target non-traded inflation. Also, the stabilisation of interest rates seems to be welfare enhancing under CPI targeting while increasing preference towards exchange rate stabilisation results in less welfare losses under ET.

In terms of the ranking of monetary policy regimes, the results are generally robust to changes in some parameters such as openness, price stickiness and elasticity of substitution. The analysis shows that increasing openness tends to increase the real exchange rate fluctuations under ET regime. On the other hand, reducing price stickiness has the effect of propagating higher real exchange rate volatility under ET than in other regimes. Also, reducing the elasticity of substitution does not change the response patterns of most variables and the performance of different monetary policy rules.

The results generally suggest that the central bank can reduce macroeconomic volatility by targeting CPI inflation. However, this stabilisation comes at the cost of higher exchange rate volatility. This implies that when the central bank responds to external shocks, it should consider the economy’s greater vulnerability to exchange rate fluctuations. The evidence from welfare analysis suggest that a small open economy exposed to the volatile commodity terms of trade shocks can reduce welfare losses by targeting CPI inflation. The central bank that want to minimise welfare losses while stabilising output, should target non-traded inflation. Although CIT seems to stabilise most variables and enhance welfare, it is not entirely the best policy in all respects because of some volatility trade-offs and constraints by policy maker’s preferences.
References


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7.1 Appendix

Appendix A

A. Household optimisation

\[
\begin{align*}
\text{Max } U &= E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{C_t^{1-\sigma}}{1-\sigma} - \eta \frac{L_t^{1+\psi}}{1+\psi} \right) \\
\text{s.t. } P_t C_t &= W_t L_t + \Pi_t + D_t - E_t(Q_{t+1} D_{t+1}) \\
\end{align*}
\]

(A.1)

First order conditions:

\[
\frac{dL}{dC_t} = \frac{1-\sigma}{1-\sigma} (C_t)^{-\sigma} + \lambda_t P_t = 0
\]

(A.2)

\[
C_t^{-\sigma} = -\lambda_t P_t
\]

(A.3)

\[
C_{t+1}^{-\sigma} = -\lambda_{t+1} P_{t+1}
\]
Dividing equation (A.3) by (A.2) gives:

$$\left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} = -\frac{\lambda_{t+1} P_{t+1}}{-\lambda_t P_t}$$  \hspace{1cm} (A.4)

$$\frac{dL}{dL_t} = -\frac{1 + \psi}{1 + \psi} L_t^\psi - \lambda_t W_t$$

$$-\frac{\eta L_t^\psi}{W_t^\beta} = \lambda_t$$  \hspace{1cm} (A.5)

Substituting equation (A.2) into (A.5):

$$C_t^\sigma \eta L_t^\psi = \frac{W_t}{P_t}$$  \hspace{1cm} (A.6)

$$\frac{dL}{dD_t} = \lambda_t E_t (Q_{t+1}) - \beta \lambda_{t+1} = 0$$

$$\frac{E_t (Q_{t+1})}{\beta} = \frac{\lambda_{t+1}}{\lambda_t}$$  \hspace{1cm} (A.7)

Combining (A.7) and (A.4) gives:

$$\beta \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \left( \frac{P_t}{P_{t+1}} \right) = E_t (Q_{t+1})$$  \hspace{1cm} (A.8)

Using and substituting in equation (A.8) $E_t (Q_{t+1}) = R_t^{-1}$ gives:

$$\beta R_t E_t \left( \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \left( \frac{P_t}{P_{t+1}} \right) \right) = 1$$  \hspace{1cm} (A.9)

**B. Real exchange rate and commodity terms of trade**

$$Q_t = \frac{\varepsilon_t P_t}{P_t^*}$$  \hspace{1cm} (B.1)

The law of one price is assumed to hold for both imports and exports such that:

$$P_{xt} = \frac{P_{xt}^*}{\varepsilon_t}$$  \hspace{1cm} (B.2)

$$P_{tt} = \frac{P_{tt}^*}{\varepsilon_t}$$  \hspace{1cm} (B.3)

Substituting $P_t$ and $P_t^*$ from equations (9) and (31) in the real exchange rate equation gives:

$$Q_t = \frac{\varepsilon_t P_{Nt}^* P_{Tt}^{1-\alpha}}{P_{Nt}^* P_{Tt}^{1-\alpha}}$$  \hspace{1cm} (B.4)
Substituting $P_{Nt}$, $P_{Tt}$ and $P_{Nt}^*$ from equations (24), (29) and (34), gives:

$$Q_t = \left( \frac{X_t}{A_{Nt}} \right)^{\alpha} \left( \frac{P_{Tt}}{P_{Nt}} \right)^{1-\alpha} \left( \frac{1}{MC_{Nt}} \right)^{\alpha}$$

(B.5)

Since $\varepsilon_t P_{Xt} = P_{Xt}^*$, substituting into equation (B.5) gives:

$$Q_t = \left( \frac{X_t}{A_{Nt}} \right)^{\alpha} \left( \frac{P_{Xt}}{P_{Nt}} \right)^{1-\alpha} \left( \frac{1}{MC_{Nt}} \right)^{\alpha}$$

(B.6)

If $\alpha = \alpha^*$,

$$Q_t = \left( \frac{A_{Xt}^* A_{Nt}^* P_{Xt}^*}{A_{lt}^* A_{Nt}^* P_{lt}^*} \right)^{\alpha} \left( \frac{1}{MC_{Nt}^*} \right)^{\alpha}$$

(B.7)

C. International risk sharing and uncovered interest parity

$$\beta E_t \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \left( \frac{P_t}{P_{t+1}} \right) = E_t (Q_{t+1})$$

(C.1)

Since $E_t (Q_{t+1}) = \frac{1}{R_t}$, equation (C.1) gives:

$$\beta E_t \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \left( \frac{P_t}{P_{t+1}} \right) = \frac{1}{R_t}$$

(C.2)

International risk sharing implies that:

$$\beta E_t \left( \frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma} \left( \frac{\varepsilon_t P_t^*}{\varepsilon_{t+1} P_{t+1}^*} \right) = \frac{1}{R_t}$$

(C.3)

Equating domestic and foreign consumption:

$$\beta E_t \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \left( \frac{P_t}{P_{t+1}} \right) = \beta E_t \left( \frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma} \left( \frac{\varepsilon_t P_t^*}{\varepsilon_{t+1} P_{t+1}^*} \right)$$

(C.4)

Solving:

$$C_t^{-\sigma} = \left( \frac{C_{t+1} C_t^*}{C_{t+1}^* C_t} \right)^{-\sigma} \left( \frac{P_t}{P_{t+1}^*} \right) \left( \frac{\varepsilon_{t+1} P_{t+1}}{\varepsilon_t P_t^*} \right)$$

(C.5)

But $Q_t = \frac{\varepsilon_t P_t}{P_t}$; and $Q_{t+1} = \frac{\varepsilon_{t+1} P_{t+1}}{P_{t+1}}$, substituting:

$$C_t = \left( \frac{C_{t+1}}{C_{t+1}^*} \right)^{\frac{1}{Q_t}} \frac{1}{Q_t^{\frac{1}{Q_t}}} C_t^*$$

(C.6)
As shown in Gali and Monacelli (2005) iterating this results in:

\[ C_t = \Omega Q_t^{\frac{1}{\sigma}} C_t^* \]  

(C.7)

where \( \Omega \) is a constant that represents initial asset positions.

Under complete markets, the uncovered interest parity condition can be derived as follows:

\[ \beta E_t \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \left( \frac{P_t}{P_{t+1}} \right) = E_t Q_{t+1} = \frac{1}{R_t} = \beta E_t \left( \frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma} \left( \frac{\varepsilon_t P_t^*}{\varepsilon_{t+1} P_{t+1}^*} \right) \]  

(C.8)

Also,

\[ \beta E_t \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \left( \frac{P_t^*}{P_{t+1}^*} \right) = \frac{1}{R_t^*} \]  

(C.9)

Substitute (C.8) into (C.9) gives:

\[ \frac{1}{R_t^*} \frac{\varepsilon_t}{E_t \varepsilon_{t+1}} = \frac{1}{R_t} \]  

(C.10)

Substituting \( \frac{1}{R_t^*} \) for \( E_t Q_{t+1} \) in equation (C.10), gives:

\[ E_t Q_{t+1} = \frac{1}{R_t^*} \frac{\varepsilon_t}{E_t \varepsilon_{t+1}} \]  

(C.11)

From (C.1):

\[ E_t Q_{t+1} R_t = 1 \]  

(C.12)

From (C.11):

\[ E_t Q_{t+1} R_t \frac{\varepsilon_{t+1}}{\varepsilon_t} = 1 \]  

(C.13)

Subtracting (C.12) from (C.13) lead to:

\[ E_t Q_{t+1} (R_t - R_t^* \frac{\varepsilon_{t+1}}{\varepsilon_t}) = 0 \]  

(C.14)

Log-linearising around the steady state:

\[ r_t - r_t^* = E_t \Delta \varepsilon_{t+1} \]  

(C.15)

Combining equation (C.15) with the real exchange rate equation \( q_t = \varepsilon_t + p_t - p_t^* \) and rearranging gives:

\[ E_t \Delta q_{t+1} = (r_t + E_t \pi_{t+1}) - (r_t^* + E_t \pi_{t+1}^*) \]  

(C.16)
D. Domestic price setting

Optimisation problem for optimising firms is:

\[
\text{Max } \sum_{t=0}^{\infty} (\theta_N)^k E_t \left\{ Q_{t+k} Y_{t+k} \left( \frac{P_{Nt}^{\text{new}}}{P_{Nt+k}} - MC_{Nt+k}^n \right) \right\} \text{ s.t. } Y_{t+k} \leq \left( \frac{P_{Nt}^{\text{new}}}{P_{Nt+k}} \right)^{-\varepsilon} (C_{Nt+k} + C_{Nt+k}^n)
\]

(D.1)

where \( MC_{Nt+k}^n \) is the nominal marginal cost and \( \theta_N^k E_t Q_{t+k} \) is the effective stochastic discount factor. Substituting \( Y_{t+k} \) and expanding:

\[
\mathcal{L} = \sum_{t=0}^{\infty} (\theta_N)^k E_t \left\{ Q_{t+k} \left( \frac{P_{Nt}^{\text{new}}}{P_{Nt+k}} \right)^{1-\varepsilon} (C_{Nt+k} + C_{Nt+k}^n) - \left( \frac{P_{Nt}^{\text{new}}}{P_{Nt+k}} \right)^{-\varepsilon} (C_{Nt+k} + C_{Nt+k}^n) MC_{Nt+k}^n \right\}
\]

(D.2)

\[
\frac{d\mathcal{L}}{dP_{Nt}^{\text{new}}} = \sum_{t=0}^{\infty} (\theta_N)^k E_t \left\{ \frac{Q_{t+k} (1 - \varepsilon) \left( \frac{P_{Nt}^{\text{new}}}{P_{Nt+k}} \right)^{-\varepsilon} (C_{Nt+k} + C_{Nt+k}^n) + \varepsilon \left( \frac{P_{Nt}^{\text{new}}}{P_{Nt+k}} \right) (C_{Nt+k} + C_{Nt+k}^n) MC_{Nt+k}^n}{1} \right\} = 0
\]

(D.3)

Substituting the value of \( Y_{t+k} \), and factorising gives the first order condition:

\[
\sum_{t=0}^{\infty} (\theta_N)^k E_t \left\{ Q_{t+k} Y_{t+k} \left( \frac{P_{Nt}^{\text{new}}}{P_{Nt+k}} - \frac{\varepsilon}{\varepsilon - 1} MC_{Nt+k}^n \right) \right\} = 0
\]

(D.4)

Using the fact that \( E_t (Q_{t+1}) = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \left( \frac{P_t}{P_{t+1}} \right) \) which implies that \( (Q_{t+k}) = \beta^k E_t \left( \frac{P_t}{P_{t+k}} \right) \left( \frac{C_{Nt+k}}{C_t} \right)^{-\sigma} \), gives:

\[
\sum_{t=0}^{\infty} (\theta_N \beta)^k E_t \left\{ P_{Nt}^{\text{new}} \left( \frac{C_{t+k}^{-\sigma}}{P_{Nt+k}} P_{t+k}^{-1} C_{t+k}^{-\sigma} Y_{t+k} \left( \frac{P_{Nt}^{\text{new}}}{P_{Nt+k}} - \frac{\varepsilon}{\varepsilon - 1} MC_{Nt+k}^n \right) \right) \right\} = 0
\]

(D.5)

Since \( P_{Nt} C_t^{-\sigma} \) are known at time, we can take them off. Thus:

\[
\sum_{t=0}^{\infty} (\theta_N \beta)^k E_t \left\{ P_{Nt+k}^{-1} C_{Nt+k}^{-\sigma} Y_{t+k} \left( \frac{P_{Nt}^{\text{new}}}{P_{Nt+k}} - \frac{\varepsilon}{\varepsilon - 1} MC_{Nt+k}^n \right) \right\} = 0
\]

(D.6)

Using the definition of real marginal costs \( MC_{t+k}^R = \frac{MC_{Nt+k}^n}{P_{Nt+k}} \), substituting into equation (D.6), dividing by \( \frac{P_{Nt-1}}{P_{Nt+k}} \) and factorising \( P_{Nt-1} \) gives:

\[
\sum_{t=0}^{\infty} (\theta_N \beta)^k E_t \left\{ C_{t+k}^{-\sigma} Y_{t+k} \frac{P_{Nt-1}}{P_{Nt+k}} \left( \frac{P_{Nt}^{\text{new}}}{P_{Nt-1}} - \frac{\varepsilon}{\varepsilon - 1} MC_{Nt+k}^R \left( \frac{P_{Nt+k}}{P_{Nt-1}} \right) \right) \right\} = 0
\]

(D.7)
Where $\frac{e}{t-1}$ is the markup. Using the geometric sum formula $\sum_{t=0}^{\infty} (\theta_N \beta)^k = \frac{1}{1 - \theta_N \beta}$ and log-linearising, gives:

$$p_{Nt}^{new} = p_{Nt-1} + \sum_{t=0}^{\infty} (\theta_N \beta)^k \left\{ E_t \pi_{Nt+1} + (1 - \theta_N) E_t m c^R_{Nt+k} \right\} \quad (D.8)$$

Rewriting equation (D.8) after splitting the equation into two parts $t$ and $t + 1$ to $\infty$, gives:

$$p_{Nt}^{new} = p_{Nt-1} + \pi_{Nt} + (1 - \theta_N) m c^R_{Nt} + (\theta_N \beta) \sum_{t=0}^{\infty} (\theta_N \beta)^k \left\{ E_t \pi_{Nt+k} + (1 - \theta_N) E_t m c^R_{Nt+k} \right\} \quad (D.9)$$

Using equation (D.8) to substitute the last term of equation (D.9) and rearranging gives:

$$p_{Nt}^{new} - p_{Nt-1} = (\theta_N \beta) \left\{ E_t \pi_{Nt+1} - p_{Nt-1}^{new} \right\} + \pi_{Nt} + (1 - \theta_N) m c^R_{Nt} \quad (D.10)$$

Substituting $p_{Nt}^{new} - p_{Nt-1}$ of equation (D.9) into equation (D.10) and solving for $\pi_{Nt}$:

$$\pi_{Nt} = \beta E_t \pi_{Nt+1} + \lambda_{Nt} m c^R_{Nt} \quad (D.11)$$

where $\lambda_{Nt} = \frac{(1 - \theta_N)(1 - \theta_N)}{\theta_N}$ and $\theta_N$ is the stickiness parameter.
Figure 4: Impulse responses to commodity terms of trade shocks: high openness
Figure 5: Impulse responses to commodity terms of trade shock: no price stickiness
Figure 6: Impulse responses to commodity terms of trade shock: low elasticity of substitution between traded and non traded goods
Table 4: Sensitivity tests on volatality

<table>
<thead>
<tr>
<th></th>
<th>High openness</th>
<th>No price stickiness</th>
<th>Low elasticity of substitution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CIT</td>
<td>NTIT</td>
<td>ET</td>
</tr>
<tr>
<td>Output</td>
<td>0.11</td>
<td>0.35</td>
<td>0.22</td>
</tr>
<tr>
<td>Non-traded output</td>
<td>0.13</td>
<td>0.32</td>
<td>0.15</td>
</tr>
<tr>
<td>Exported output</td>
<td>0.51</td>
<td>0.19</td>
<td>0.61</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.25</td>
<td>0.43</td>
<td>0.45</td>
</tr>
<tr>
<td>Nominal exchange rate</td>
<td>0.20</td>
<td>0.18</td>
<td>0.03</td>
</tr>
<tr>
<td>Real exchange rate</td>
<td>0.14</td>
<td>0.07</td>
<td>0.09</td>
</tr>
<tr>
<td>Non-traded prices</td>
<td>0.29</td>
<td>0.02</td>
<td>0.25</td>
</tr>
<tr>
<td>Non-traded inflation</td>
<td>0.32</td>
<td>0.04</td>
<td>0.34</td>
</tr>
<tr>
<td>CPI inflation</td>
<td>0.49</td>
<td>0.25</td>
<td>2.34</td>
</tr>
<tr>
<td>Interest rates</td>
<td>2.67</td>
<td>0.02</td>
<td>0.09</td>
</tr>
</tbody>
</table>

CIT is CPI inflation targeting, NTIT is non-traded inflation targeting and ET is exchange rate targeting.

Table 5: Sensitivity tests on welfare evaluations

<table>
<thead>
<tr>
<th>Weights on central bank loss function</th>
<th>Welfare losses</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>High openness</td>
</tr>
<tr>
<td></td>
<td>IT</td>
</tr>
<tr>
<td>λx 0.5  λy 0.5  λr 0.5  λc 0.5</td>
<td>0.36</td>
</tr>
<tr>
<td>1 0.5  λy 0.5  λr 0.5</td>
<td>0.32</td>
</tr>
<tr>
<td>1.5 0.5  λy 0.5  λr 0.5</td>
<td>0.17</td>
</tr>
<tr>
<td>2 0.5  λy 0.5  λr 0.5</td>
<td>0.62</td>
</tr>
<tr>
<td>0.5 1  λy 0.5  λr 0.5</td>
<td>0.16</td>
</tr>
<tr>
<td>0.5 1.5  λy 0.5  λr 0.5</td>
<td>0.05</td>
</tr>
<tr>
<td>0.5 2  λy 0.5  λr 0.5</td>
<td>0.03</td>
</tr>
<tr>
<td>0.5 0.5  λy 1  λr 0.5</td>
<td>0.12</td>
</tr>
<tr>
<td>0.5 0.5 1.5  λr 0.5</td>
<td>0.33</td>
</tr>
<tr>
<td>0.5 0.5 2  λr 0.5</td>
<td>0.11</td>
</tr>
<tr>
<td>0.5 0.5 0.5  λc 0.5</td>
<td>0.06</td>
</tr>
<tr>
<td>0.5 0.5 0.5 1.5  λc 0.5</td>
<td>0.08</td>
</tr>
<tr>
<td>0.5 0.5 0.5 2  λc 0.5</td>
<td>0.08</td>
</tr>
</tbody>
</table>

CIT is consumer price index inflation targeting, NTIT is non-traded inflation targeting and ET is exchange rate targeting.